

Interesting, Surprising and perhaps Amusing Results in Nuclear Physics Calculations over the years

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Abstract

We have compiled some sectors of works by L. Zamick and collaborators which we hope will be of interest to the reader. Some of the results which we, at least, have found amusing often popped up unexpectedly in what were apparent routine calculations. We feel that the sections we have made can be of importance in farthing the field but our main criteria are that the results will have some educational value and that they add a bit of zest to our field.

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Nuclear Compressibility

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Abstract

A simple formula is obtained, which relates the nuclear compressibility to the binding energy per particle, the mean kinetic energy per particle, and the power of the density appearing in the two body interaction.

In this work we wish to calculate the compressibility of a nucleus, assuming an interaction which compressibility of a nucleus, assuming an interaction which correctly reproduces the total binding energy and the mean square radius. Such that an interaction requires at least two parameters in order to obtain these two properties, but we shall use a three-parameter interaction

$$-\alpha\delta(r) + \gamma\rho^\sigma(R)\delta(r)$$

This a familiar density dependent interaction, not unsimilar to the one chosen by Moszkowski [1], except that he fixes the power of the density, σ , to a value of 2/3 and he includes finite range corrections. Vautherin and Brink [2] have $\sigma=1$.

Here, on the contrary, the parameter σ is kept as a variable, for we wish to study what influence it has on the nuclear compressibility.

We assume that we can restrict ourselves to harmonic-oscillator wave functions, characterized by only one parameter, the oscillator length b . In that case we can by dimensional analysis, write the expression for the total binding energy as

$$E = -E_B = A/b^2 + B/b^3 + C/b^{(3+3\sigma)}$$

where A , B , and C are independent of b . The first term is the kinetic energy and is given by the oscillator formula

$$A/b^2 = (\hbar^2 / (2mb^2)) \sum 2n + l + 3/2$$

Since we are at equilibrium

$$\frac{dE}{db} = 0$$

$$0 = -2A/b^3 - 3B/b^4 - (3 + 3\sigma) C/b^{(4+3\sigma)}$$

We can regard the above as two equations in the two unknowns B and C . We obtain

$$B = -b^3 (1 + \sigma) E_B / \sigma - \langle T \rangle b^3 (1 + 3\sigma) / 3\sigma$$

$$C = b^{(3+3\sigma)} E_B / \sigma + \langle T \rangle b^{(3+3\sigma)} / 3\sigma$$

evaluated at the equilibrium value of b and E_B .

The nuclear compressibility K , is defined as

$$K = (1/sA) b^2 \frac{d^2 E}{db^2}$$

We thus obtain an expression for K totally in terms of the binding energy per particle and mean kinetic energy of a particle (both positive quantities)

$$K = (1/sA) [\langle T \rangle + qE_B + \sigma (3 \langle T \rangle + qE_B)]$$

This is our main result. Note that the compressibility is linear in the power of the density σ and increases with increasing σ .

The above is an obvious explanation of Bethe's observation [3] that the Vautherin Brink interaction [2] ($\sigma=1$), yields a considerably larger compressibility than the interaction he used.

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Mass Parameter of the Breathing Mode State

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Abstract

A simple identity is used to show that when harmonic oscillator wave functions are used, Inghs formula for the mass parameter of the breathing mode state yields the classical result.

We note the following relationship obeyed by the operator r^2 (square radius) when Slater determinants with harmonic oscillator radial wave functions are used

$$\sum_n \left| \langle 0 | \sum r^2 | n \rangle \right|^2 = b^2 \langle 0 | \sum r^2 | 0 \rangle$$

In the above b is the oscillator length parameter, $|0\rangle$ is the ground state of a closed shell and $|n\rangle$ is an excited state. We need only consider excited states of the type $\left| \alpha_{n+1}^{j+1} \alpha_n^j \right|^{J=0}$, that is one-particle-one hole states which differ by a node and couple to $J=0$. Working out both sides we get

$$\sum (sl+1)(n+1)(n+l+3/2) = \sum (2l+1)(2n+l+3/2)$$

$$n = 0, 1, 2, \dots$$

On the left we sum only over those occupied states nlj which are such that the state $n+1, lj$ is above the Fermi sea; on the right we sum over all occupied states.

The above can be derived by noting that the harmonic oscillator spectrum is such that $(E_n - E_0)$, the energy difference, is constant $-2\hbar\omega$. Hence the energy independent sum rule for $E(0)$ transitions is proportional to the energy weighted sum rule.

As noted by Werntz and Uberall [2], if one considers classically a sphere of mass M and root mean square radius η which undergoes breathing mode oscillations then the kinetic energy is $1/2 M\dot{\eta}^2$. We wish to show that the

same result is obtained in the Inglis model [1] using harmonic oscillator wave functions.

Let us first define an auxiliary mass parameter

$$M_b = 2\hbar^2 \sum_n \left| \left\langle n \left| \frac{\partial}{\partial b} \right| 0 \right\rangle \right|^2 / (E_f - E_i)$$

A harmonic oscillator wave function has the structure $N(b)f(r/b)$, where N is the normalization, and b is the oscillator length. We have

$$\frac{\partial}{\partial b} N(b) f(r/b) = \frac{dN}{db} f - r/bN \frac{\partial f}{\partial r}$$

The first term does not contribute because of the orthogonality of $|n\rangle$ and $|0\rangle$. Hence we get

$$M_b = (2\hbar^2) / b^2 \sum_n \left| n \left\langle \sum_i r_i \frac{\partial}{\partial r_i} \right| 0 \right\rangle \right|^2$$

The quantity $r \frac{\partial}{\partial r}$ can be written as a commutator involving the shell model Hamiltonian

$$r \frac{\partial}{\partial r} = (-m/2\hbar^2) [H_{sm}, r^2] - 3/2$$

the last term does not contribute. We also can make the simplification $\langle n [H_{sm}, r^2] 0 \rangle = (E_f - E_i) \langle n | r^2 | 0 \rangle$. Finally, but replacing $(E_f - E_i)$ by $2\hbar\omega = (2\hbar^2) / (mb^2)$ we get

$$M_b = 2\hbar^2 \sum_n \left| \left\langle n \left| \frac{\partial}{\partial b} \right| 0 \right\rangle \right|^2 / b^4$$

But we are interested in the mass parameter M_{rms} , in which the root mean square radius η is the dynamical variable. We get

$$M_{rms} = \left(\frac{\partial b}{\partial \eta} \right)^2 M_b = (b^2 M_b) / \left(\langle 0 | \sum r^2 | 0 \rangle / A \right)$$

Hence, we get

$$M_{rmsA} = mA \left\{ \frac{\sum_b |\langle n | \sum r^2 | 0 \rangle|^2}{b^2 \langle 0 | \sum r^2 | 0 \rangle} \right\}$$

But from the relationship at the beginning of this paper the quantity in brackets is one. Hence $M_{rms} = mA$.

For completeness we note the well known equation of motion for the breathing mode state

$$\frac{1}{2} M_{rms} \dot{\eta}^2 + \frac{1}{2} AK (\eta - \eta_o)^2 / \eta_o^2 = E$$

where K is the nuclear compressibility. The energy of the breathing mode state is

$$(\hbar\omega)_B = (AK / (M_{rms} \eta_o^2))^{1/2} = \sqrt{K / (m \eta_o^2)}$$

One often takes $\eta_o^2 = \frac{3}{5}r_o^2A^{2/3}$ with $r_o=1.2$ fm. I would like to thank J. Speth and A. Fassler for their interest and hospitality.

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Isospin Mixing of the $1+$ States in $^{12}\text{C}^*$

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Abstract

The isospin mixing of the lowest two 1^+ states in ^{12}C is here considered. A nuclear charge symmetry breaking interaction (CSB) which was previously fitted to help explain the Nolen-Schiffer anomaly, goes in the right direction for this problem, but is not as important as in the N-S problem. An important effect is mixing due to the one body Coulomb field. This is shown to be approximately proportional to $(r^2)_{P_{1/2}} - (r^2)_{P_{3/2}}$, a quantity which is very sensitive to the details of the one body nuclear potential.

The total off-diagonal matrix elements are 170 keV with the DME and 92 keV with the SKII interaction. The DME can reproduce the experimental value obtained by Lind et al., while the SKII interaction reproduces the one obtained by Adelberger et al.

Why are the DME and SKII interactions so different? Looking in table 2 we note that they differ by 62 keV in the off-diagonal matrix element of one body coulomb field. Thus the quantity which is conceptually the simplest thing entering into this calculation is giving the largest ambiguity.

By following an approximate procedure of Lane Martorelli and Zamick [10], we can relate the matrix element of the Coulomb potential to the square radius of a shell model orbit. We approximate U_c by its value in the interior of a uniform sphere

$$U_c \approx \frac{3Ze^2}{2R} \left(1 - \frac{r^2}{3R^2}\right) \quad (6)$$

Thus

$$\langle T = 1 | U_c | T = 0 \rangle = \frac{Ze^2}{4R^3} \left[\langle r^2 \rangle_{\pi p_{1/2}} - \langle r^2 \rangle_{\pi p_{3/2}} \right] \quad (7)$$

We now see why we are getting large differences between DME and SKII interaction. Whereas for the $p_{3/2}$ proton orbit the difference $\langle r^2 \rangle_{DME} - \langle r^2 \rangle_{SKII} -$

0.126 fm² or 1.7%, the corresponding difference for the $p_{1/2}$ proton orbit is much bigger, 0.962 fm² or 10%.

Why is there such a large difference between DME and SKII? A possible explanation is that the $p_{1/2}$ orbit is bound by only 5.74 MeV for DME, but for SKII interaction the value is 7.15 MeV. This certainly goes in the right direction.

We examine this possibility in more detail by making the following list.

$$\text{Define } \Delta = \left[\langle r^2 \rangle_{p_{1/2}} - \langle r^2 \rangle_{p_{3/2}} \right]_{\text{proton}}$$

$$\Delta [fm^2] \langle T = 1 | U_c | T = 0 \rangle = \frac{Ze^2 \Delta}{4R^3} [keV]$$

<i>DME</i>	2.172	178
<i>WS[DME]</i>	1.844	151
<i>SKII</i>	1.336	109
<i>WS[SKII]</i>	1.347	109

In the above WS[DME] stands for the Woods-Saxon parameters which give the same single particle energies as DME etc. Note that Δ is the same for the SKII interaction as for WS[SKII]. However, there is a discrepancy between DME WS [DME] such that when all is said and done the DME will give a larger off-diagonal matrix element. Perhaps the most interesting comment from the above list is that we get a change of 45 keV in going from WS [DME] to WS [SKII]. This is, by simply demanding the $p_{1/2}$ orbit be bound by 5.74 MeV rather than 7.15 MeV, we are getting a change which is starting to be comparable to the difference in the values which have been cited by the two experimental groups.

In conclusion we note that our results are not inconsistent with the currently available experimental data; however, those data have large error bars and there is some disagreement between two groups. The phenomenological charge symmetry breaking interaction which helps explain the Nolen-Schiffer anomaly also helps to enhance the isospin mixing matrix element. The results depend sensitively upon the square radii of the single particle orbits. Indeed two different Hartree-Fock models give different results for these quantities, and it is hard at present to favor one model over the other. Further work along these lines i.e. learning what are the correct single particle energies for the shell model orbits, is suggested.

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Nuclear vibrations with a zero-range interaction and the multipole condition

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Abstract

Isoscalar monopole, quadrupole, and octopole states are calculated in closed shell nuclei. A delta interaction is used, its strength determined by the multipole condition, namely that the mean single-particle-single-hole potential energy difference is equal to the corresponding kinetic energy difference. This interaction is used to obtain particle-hole matrix elements appropriate for a random-phase approximation calculation. The strengths given by different multipole conditions are different but (except $L=0$) they appear to approach each other as the mass number A becomes large. The monopole mode has already collapsed somewhat before the strength implied by the quadrupole condition is reached. Some of the calculations were repeated using a zero-range Skyrme interaction. We observe a very high degeneracy in our calculation which we are able to explain in terms of L-S coupling and the fact that the particle-hole matrix element of a delta interaction is of the form $\int f_{P_A H_A}(r) f_{P_B H_B}(r) r^2 dr$.

IV. EXPLANATION OF HIGH DEGENERACY IN L-S

Coupling Although a j-j coupling basis was used for these calculations, it should be noted that the interaction we use is central, and we are not introducing a one body spin orbit interaction. This means that the quantum numbers L and S are good. This manifests itself in the fact that most of the states that we obtain in an RPA diagonalization have vanishing $B(EL)$'s to the ground state. These correspond to states with $S \neq 0$.

Furthermore, if we choose the single particle energies to be those of a harmonic oscillator (this was described in more detail earlier as SPII), we find that even fewer states than are permitted by L-S coupling have finite $B(EL)$ transition to ground states.

Furthermore, we observe that these states which have vanishing B(EL)'s have eigenvalues which are some multiples of $\hbar\omega$. For the quadrupole states there is a high degeneracy of $2\hbar\omega$; in the octopole case at $1\hbar\omega$ and $3\hbar\omega$. Take for example $L=2^+$ states in ^{16}O . We find that there are 4 states which are degenerate at exactly $2\hbar\omega$. These states have zero B(E2) strength to the ground state. This degeneracy can be partly explained by looking at the expression for the particle-hole interaction in L-S coupling which is given in Appendix A. The particle-hole states forming the 2^+ states in ^{16}O can be divided into 3 classes.

$$\begin{array}{llll} \text{Class(1)} & L \neq 2 & S = 1 & (0p^{-1}0f) L = 3 \quad (0p^{-1}1p) L = 1 \\ \text{Class(2)} & L = 2 & S = 0 & (0s^{-1}0d) (0p^{-1}0f) \quad (0p^{-1}1p) \\ \text{Class(3)} & L = 2 & S = 1 & (0s^{-1}0d) (0p^{-1}0f) \quad (0p^{-1}1p) \end{array}$$

The 3j symbol

$$\begin{array}{ccc} l_h & l_p & \bar{L} \\ 0 & 0 & 0 \end{array}$$

vanishes unless $l_h + l_p + L$ is even. Hence all the matrix elements in class 1 will vanish. This explains two of the four states at $2\hbar\omega$.

We next note that for every matrix element in class 2 that vanishes there must be a corresponding one in class 3 that vanishes. This is because the entire spin dependence is contained in the factor $(1 - 4\delta_{S,o} \delta_{T,o} + 2x\delta_{S,o} - 2x\delta_{T,o})$. Since we are setting $x=0$, this factor will be 1 for $S=1$ and -3 for $S=0$. The particle-hole matrix elements in class 2 are -3 times those in class 3.

Thus two of the four degenerate states are from class 1, one from class 2, and one from class 3. The ones from class 2 and class 3 are expected to have the same radial structure, differing only in the spins. Only class 2 states can have finite B(E2)'s. This is because the E2 operator $\sum \gamma^2 y_2$ has no spin dependence and therefore cannot connect spin one to spin zero. Thus, in this example we are left with one nontrivial degeneracy (class 2) to explain, as well as the vanishing B(E2).

To explain this, we note that the L-S coupling expression for the particle-hole matrix element (given in Appendix A) has the structure

$$\langle [p'h'^{-1}]^{LST} V [ph^{-1}]^{LST} \rangle = \int f'_k(r) f_k(r) r^2 dr$$

where from here on we use the symbol k to designate ph .

Let us denote the class 2 state at $2\hbar\omega$ by ψ and expand it in terms of particle-hole components (we limit ourselves to TDA states in this discussion).

$$\psi = \sum z_k |k\rangle$$

We Expect

$$(\psi V_{ph} \psi) = 0$$

This can be achieved by demanding

$$\sum_k f_k(r) z_k = 0$$

Let us first consider the schematic approximation in which the integral $\int R_{n_1 l_1}(r) R_{n_2 l_2}(r) R_{n_3 l_3}(r) R_{n_4 l_4}(r)$ is replaced by a constant.⁴ In this case the particle hole matrix element has the structure $\langle k' V_{ph} k \rangle = g'_k g_k$, where g goes not depend on r . We can regard g_k as elements of a vector of dimension D , where D is the number of particle-hole states for a given LST configuration, e.g., in the above example, ^{16}O $L=2$, $S=0$, $T=0$ we have $D=3$. The quantities z_k are also elements of a vector of dimension D , and the condition $\sum_k z_k g_k = 0$ simply means that the vector $\{z\}$ is orthogonal to the vector $\{g\}$.

There are clearly $(D-1)$ vectors $\{z\}$ which are orthogonal to $\{g\}$. Each of these $(D-1)$ vectors will have a vanishing particle-hole matrix element. Thus the class 2 degeneracy at $2\hbar\omega$ will be $(D-1)$. This will also be for the class 3 degeneracy. Thus for $L=2$, $S=0$, and $T=0$ states in ^{16}O we expect a twofold degeneracy from class 2. This was confirmed by a calculation. We obtain the same degeneracy in class 3.

We now consider the case where $f_k(r)$ is not approximated by a constant. We note that $f_k(r)$ is proportional to the product of two harmonic oscillator radial wave functions $R_n(r)$, $R_h(r)$. Using the Variable $x=r/b$ we note that the product can be written in the form of an exponential times a polynomial

$$f_{ph}(r) = e^{-x^2} X^\sigma \sum_{N_{min}(ph)}^{N_{max}(ph)} a_n(ph) X^{2n}$$

where $\sigma=0$ or 1 . Let N_{min} =minimum of all $N_{min}(ph)$ and let N_{max} = maximum of all $N_{max}(ph)$. To ensure that $\sum_k f_k(r) z_k = 0$ we demand that each coefficient of $X^{2n+\sigma}$ vanishes. That is, $\sum a_n(k) z_k = 0$. This leads to Δ conditions where $\Delta = N_{max} - N_{min} + 1$. We now have Δ vectors, the n th one of which is $[a_n(1), a_n(2), \dots, a_n(D)]$. The vector $\{z(k)\}$ has to be orthogonal to all of these. There are clearly $(D - \Delta)$ such vectors $\{z(k)\}$. Hence class 2 degeneracy is $(D - \Delta)$.

In our example ($L=2$, $S=0$, and $T=0$ in ^{16}O) the polynomial has terms in x^2 and x^4 . Thus $N_{min}=1$, $N_{max}=2$, and hence $\Delta=2$. Thus the degeneracy $(D - \Delta)$ is one.

In a major shell n , l , N ($N=2n+1l$) the lowest power that appears in a polynomial is x^1 ; the highest power is x^N . Thus if the particle is in the shell n_p , l_p , N_p , and the hole is the shell n_h , l_h , N_h , then we have

$$N_{max}(ph) = \frac{1}{2}(N_p + N_h) \text{ if } N_p + N_h \text{ is even}$$

$$N_{max}(ph) = \frac{1}{2}(N_p + N_h - 1) \text{ if } N_p + N_h \text{ is odd}$$

$$N_{min}(ph) = \frac{1}{2}(l_p + l_h) \text{ if } l_p + l_h \text{ is even}$$

$$N_{min}(ph) = \frac{1}{2}(l_p + l_h - 1) \text{ if } l_p + l_h \text{ is odd}$$

Just to give another example, consider the $L=2$ and $T=0$ states in ^{40}Ca . There are seventeen states in all, five in class 1, six in class 2, and six in class 3.

The entire degeneracy at $2\hbar\omega$ for the schematic model 15. Five of the states are from class 1. Since D is 6 for class 2 the degeneracy here is $D-1=5$. It is also 5 for class 3.

For the delta interaction the entire degeneracy at $2\hbar\omega$ is eleven. Five still come from class 1, leaving 3 from class 2 and 3 from class 3. We find that the quantity Δ is equal to three. The class 2 degeneracy is therefore $D - \Delta = 3$, as expected.

It is very easy to show that the $B(\text{EL})$ to ground is zero for these degenerate states.

The $B(\text{EL})$ is proportional to $\int r^L \sum_k f_k(r) z_k dr$. Since the integrand is zero, the integral will also be zero. The above is not true when we approximate the radial integral by a constant.

It should be emphasized that while part of the above argument involved spin isospin symmetry (as discussed many years ago by de Shalit and Walecka⁵), the crucial part involving class 2 degeneracies did not.

It is also worthwhile noting that for L - S coupling the expression for the particle-hole matrix element the spin-isospin factor is very simple $(1 - 4\delta_{S,o} + 2x\delta_{S,o} - 2x\delta_{T,o})$. The values of this factor for the four different modes are -3 for $S=0, T=0$; $(1-2x)$ for $S=1, T=0$; $(1+2x)$ for $S=0, T=1$; and 1 for $S=1, T=1$.

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Comparison of magnetic dipole excitations in the $f_{7/2}$ shell region with the new collective in ^{156}Gd

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Abstract

It is noted that in a single j shell $f_{7/2}$ calculations in the titanium isotopes one obtains $M1$ rates of about one single particle unit in strength to states at about 4 MeV excitation. This is systematically consistent with the recently discovered collective excitations in ^{156}Gd , and neighboring nuclei. In the single j shell case, through, the spin and orbital contributions to the $M1$ matrix element are nearly the same.

Recently a low lying magnetic dipole excitation has been discovered in ^{156}Gd at an excitation energy of 3.075 MeV with a strength $B(M1) \uparrow - (1.3 \pm 0.2) \mu_N^2$.¹ The experiment involving high resolution inelastic electron scattering with the Darmstadt linear accelerator. This was followed by the discovery, by the Darmstadt group, of low lying 1^+ states in other deformed nuclei such as ^{154}Sm , ^{154}Gd , ^{164}Dy , ^{168}Er , and ^{174}Yb .

The purpose of this Brief Report is to point out that the presence of an M1 state with an excitation energy of 3-4.5 MeV and the order of 1 single particle unit of strength may be a more widespread phenomenon. We demonstrate this by calculating M1 rates to the lowest 1^+ states of the even-even titanium isotopes, using the wave function of McCullen, Bayman, and Zamick (MBZ)^{14, 15} and Ginocchio and French.¹⁶

In a single j shell one can replace the magnetic dipole operator μ by $(g_p L_p + g_n L_n) \mu_N$, where L_p is the angular momentum operator for the protons and L_n for the neutrons. We use the same quenched g_p and g_n as were used originally,³⁴ $g_p = 1.50$ and $g_n = 0.39$. The M1 transition must be proportional to $(g_p - g_n)^2$, for otherwise, the total angular momentum operator $J - L_p - L_n$ would be able to induce an M1 transition for J=0 to J=1. This is clearly impossible.

The MBZ wave functions^{14, 15} for the titanium isotopes are of the form

$$\Psi^J = \sum_{L_p, L_n, \nu} D^J(L_p, L_n, \nu) \left[\left(f_{7/2}^2 \right)_\pi^{L_p} \left(f_{7/2}^n \right)_\nu^{L_n} \right]^J$$

where $D^J(L_p, L_n)$ is the probability amplitude that two protons couple to L_p and n neutrons couple to L_n . The parameter V is the seniority quantum number. This is relevant only to the nuclei ^{46}Ti , where for $L_n = 2$ and 4 we have states of both seniority 2 and seniority 4.

The expression for the M1 transition is

$$B(M1) \uparrow = (3/4\pi) \mu_N^2 (g_p - g_n)^2 \times \left| \sum_{LV} D^0(L, LV) D^1(L, LV) \sqrt{L(L+1)} \right|^2$$

The quantity $(g_p - g_n)$ is set equal to 1.89. The coefficients $D^{-1}(L_p, L_n)$ are contained in Ref. 17. The results for the energies and M1 rates are

	$E(\text{MeV})$	$B(M1) \uparrow (\mu_N^2)$
^{44}Ti	5.81	2.70
^{46}Ti	4.00	1.70
^{48}Ti	3.83	0.69

In ^{44}Ti we have equal numbers of neutrons and protons. All 1^+ states in the $f_{7/2}$ model have isospin T=1. The J=0⁺ ground state, of course, has T=0. This nucleus is therefore a special case.

However, for ^{46}Ti and ^{48}Ti , the 1_1^+ states have the same isospin as the ground state. It is therefore not unreasonable to compare these nuclei with ^{156}Gd and its neighbors. Considering the wide spread in mass number between the nuclei,

the behavior is remarkably similar. Perhaps then the presence of 1^+ state with single particle strength in the 3-4.5 MeV range is a widespread phenomenon.

Whether the physical interpretation of the 1^+ states in the two different regions is the same is another question. Independent of what the answer is, it will clearly be of interest to try to verify the presence over a wide range of the periodic table of 1^+ states whose energies and strengths vary systematically with mass number.

The expression for the M1 rate by Dieperink⁷ is

$$B(M1) \uparrow = \frac{3}{4\pi} (4N_\pi N_\nu) / \left((N_\pi + N_\nu) (\bar{g}_\pi - \bar{g}_\nu)^2 \mu_N^2 \right)$$

where N_π and N_ν are the number of proton and neutron boson, respectively, and \bar{g}_π and \bar{g}_ν are the boson g factors. In the single j shell model, the g factors are the same for the boson pairs $\left(f_{7/2}^2\right)^{J=2}$ as for the single particle states.

If we naively apply this formula to the $f_{7/2}$ shell, with $(\bar{g}_\pi - \bar{g}_\nu)$ set equal to $(g_p - g_n) = 1.89$ then we obtain for ^{46}Ti .

$$B(M1) \uparrow = 2.27 \mu_N^2$$

and for ^{48}Ti .

$$B(M1) \uparrow = 1.71 \mu_N^2$$

These are larger than what we calculate in the $f_{7/2}$ model.

In the single j shell model, one cannot ascribe the collective mode as a pure orbital mode. This is clear from the fact that we can replace $g_l l + g_s s$ by $g_j j$, where for $j-l-1/2$

$$g_j = (1/j)g_l + g_s/(2j)$$

while for $j-l-1/2$

$$g_j = [(l+1)/(j+1)]g_l - g_s/[2(j+1)]$$

If we use the free values of g_l and g_s , then the orbital contribution to $(g_p - g_n)$ is 0.86, and the spin contribution is 1.34.

The common choices for the renormalized values are $g_l = 1.1$ for a proton and -0.1 for a neutron, and $g_s = 0.7g_s$ (free) for the isovector term. With these values, the orbital contribution is 1.03 and the spin part is 0.94. We see that the orbit and spin contributions are nearly equal.

There is one common bond with the IBA formula,^{6,7} of course. In both expressions one has a factor $(\bar{g}_\pi - \bar{g}_\nu)^2$.

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Limited symmetry found by comparing calculated magnetic dipole spin and orbital strengths in ^4He

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Abstract

Allowing for $2\hbar\omega$ admixtures in ^4He we find that the summed magnetic dipole isovector orbital and spin strengths are equal. This indicates a symmetry which is associated with interchanging the labels of the spin with those of the orbit. Where Higher admixtures are included, the orbital sum becomes larger than the spin sum, but the sums over the low energy region are still nearly the same.

In Table I we give the total summed strength $B(M1)_{spin}$ and $B(M1)_{orbit}$ to all (nonspurious) $J = 1^+$, $T=1$ states corresponding to the operators $\vec{s}t_z$ and lt_z , respectively [as mentioned before we drop the isovector factor $5.586 - (-3.826) = 9.412$]. We do this for progressively increasing model spaces: up to $2\hbar\omega$, up to $4\hbar\omega$, and up to $6\hbar\omega$.

We perform the calculations with the spin-orbit and tensor interactions off and on.

Examining Table I we find one priori unexpected result. When we restrict the ground state correlations to $2\hbar\omega$, we find that the summed spin strengths are virtually equal to the summed orbital strengths. This is true for all four cases of (x,y), i.e., whether or not there is a tensor interaction present.

Table I. Summed spin and orbital magnetic dipole moment strengths in ^4He in units of $10^{-3}\mu_N^2$.

<i>Inter action</i>		<i>up to $6\hbar\omega$</i>		<i>up to $4\hbar\omega$</i>		<i>up to $2\hbar\omega$</i>	
<i>x</i>	<i>y</i>	<i>Spin</i>	<i>Orbit</i>	<i>Spin</i>	<i>Orbit</i>	<i>Spin</i>	<i>Orbit</i>
0	0	0.8546	0.8546	1.3357	5.1635	1.5897	7.1474
1	0	0.8569	0.8571	1.3417	5.1851	1.6211	7.2296
0	1	3.8245	3.8239	5.2346	10.937	6.0653	14.607
1	1	3.3944	3.3955	4.8288	10.554	5.6052	14.272

TABLE II. For the cases $x=0, y=0$ (central interaction, LS limit), we give the energies and $B(M1)$'s of "spin excited" and "orbit excited" states, with up to $2\hbar\omega$ admixtures.

	Energy (MeV)	B(M1) (in units of $10^{-3}\mu_N^2$)	
		Spin	Orbit
Nonspurious	3.67	0	0
	44.0	0	0
	45.3	0.855	0
	48.8	0	0
	49.2	0	0
	53.9	0	0
	56.5	0	0.835
Spurious	436.7	0	0
	436.7	0	0
	436.7	0	0
	439.3	0	13.07

We consider the case $x=y=0$. We are in the LS limit. Since the $0s^4$ closed shell has $L=0$, $S=0$, only $2\hbar\omega$ excitations with the same quantum numbers will admix into the ground state. Let us consider two particles excited from the $0s$ shell to the $0p$ shell. We can label the $2p$ - $2h$ states by $[L_\pi L_\nu]^{L=0} [S_\pi S_\nu]^{S=0}$. There are several cases to be considered:

(1) Two protons are excited. The configurations are $(p_\pi^2)^{L_\pi S_\pi} (s_\nu^2)^{L_\nu S_\nu}$. Since $L_\nu=0$ and $S_\nu=0$ and L and S are zero, we must have $L_\pi=0$ and $S_\pi=0$. So all in all we get the state $|a\rangle = (p^2)^{L_\pi=0, S_\pi=0} (s^2)^{L_\nu=0, S_\nu=0}$.

(2) Two neutrons are excited. By analogy, the configuration is $|b\rangle = (s^2)^{L_\pi=0, S_\pi=0} (p^2)^{L_\nu=0, S_\nu=0}$.

(3) A neutron and a proton are excited from the s shell to the p shell. The configuration is $[(sp)^{L_\pi S_\pi} (sp)^{L_\nu S_\nu}]^{L=0, S=0}$. There are two possibilities: $|c\rangle = [L_\pi = 1, L_\nu = 1]^{L=0} [S_\pi = 0, S_\nu = 0]^{S=0}$ and $|d\rangle = [L_\pi = 1, L_\nu = 1]^{L=0} [S_\pi = 1, S_\nu = 1]^{S=0}$.

We can form an isovector orbital excitation by applying the operation $\vec{L}_\pi - \vec{L}_\nu$ to the $J=0^+$ ground state; likewise we can form an isovector spin excitation by applying the operator $\vec{S}_\pi - \vec{S}_\nu$ to the $J=0^+$ ground state. When acting on the configurations $|a\rangle$ or $|b\rangle$, the orbital operator $\vec{L}_\pi - \vec{L}_\nu$ gives zero; likewise the spin operator $\vec{S}_\pi - \vec{S}_\nu$. That is, $(\vec{L}_\pi - \vec{L}_\nu)|L_\pi = 0, L_\nu = 0\rangle = 0$.

Let us skip to the state $|d\rangle$. Note that the orbital and spin quantum numbers are the same $L_\pi = S_\pi = 1$ and $L_\nu = S_\nu = 1$. This is enough to prove that, if this were the only state present, we would have the result $B(M1)_{spin} = B(M1)_{orbit}$.

In more detail, $(\vec{L}_\pi - \vec{L}_\nu)|d\rangle = N[L_\pi = 1, L_\nu = 1]^{L=1} [S_\pi = 1, S_\nu = 1]^{S=0}$ and $(\vec{S}_\pi - \vec{S}_\nu)|d\rangle = N[L_\pi = 1, L_\nu = 1]^{L=0} [S_\pi = 1, S_\nu = 1]^{S=1}$.

There is no reason why these states should be at the same energy and indeed they are not, but the equality of the spin and orbital strengths, provided the state $|c\rangle$ were not present, is obvious. However, the presence of the state $|c\rangle$ apparently presents a problem. The isovector spin operator $\vec{S}_\pi - \vec{S}_\nu$ will annihilate this state, whereas the isovector orbital operator $(\vec{L}_\pi - \vec{L}_\nu)$ creates the state $[L_\pi = 1, L_\nu = 1]^{L=1} [S_\pi = 0, S_\nu = 0]^{S=0}$. There should therefore be more orbital strength than spin strength. What saves the day is that this transition is spurious. In the OXBASH program [10] the spurious states are put very high in energy by adding a large constant to the single-particle energies for the

center of mass motion. We added 100 MeV for each nucleon, thus putting the spurious states in the vicinity of 400 MeV excitation energy. In Table III we show the $2\hbar\omega$ $x=0$, $y=0$ calculation in which all the $1+$, $T=1$ states are shown, both nonspurious and spurious, with the values of $B(M1)_{spin}$ and $B(M1)_{orbit}$.

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Single-particle energies and Elliott's SU(3) model

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Abstract

We address some properties of the quadrupole-quadrupole (Q.Q) interaction in nuclear studies. We here consider how to restore SU(3) symmetry even though we use only coordinate and not momentum terms. Using the Hamilton $H = \sum_i (p^2/2m + m/2\omega_i^2 r_i^2) \chi \sum_{i<j} Q(i) \cdot Q(j) - (\chi/2) \sum_i Q(i) \cdot Q(i)$ with $Q_\mu = r^2 Y_{2,\mu}$, we find that only 2/3 of the single-particle splitting $\epsilon_{0d} - \epsilon_{1s}$ comes from the diagonal term of Q.Q; the remaining 1/3 comes from the interaction of the valence nucleon with the core. The same is true in the 0f-1p shell. [S0556-2813(97)01203-X]

Here we wish to obtain Elliott's SU(3) results [2] in a shell model calculation in which only the coordinate Q.Q interaction is used. We do not wish to use the momentum-dependent terms. The latter were introduced by Elliott so that, in combination with the coordinate terms, there would be no $\Delta N=2$ admixtures, i.e., no admixture from configurations involving $2\hbar\omega$ excitations. However, in many cases we want to see the effects of such admixtures in our shell model studies. One classic problem in which $\Delta N=2$ admixtures are important is the isoscalar E2 effective charge which gets enhanced by a factor of 2 when such admixtures are allowed. There are many other problems of interest along these lines, some of which we have considered [1].

The Hamiltonian we consider is therefore

$$H = \sum_i \left(\frac{p^2}{2m} + \frac{1}{2} m \omega^2 r_i^2 \right) - \chi \sum_{i<j} Q(i) \cdot Q(j) - \frac{\chi}{2} \sum_i Q(i) \cdot Q(i)$$

where

$$Q(i)^k \cdot Q(j)^k = (-1)^k \sqrt{(2k+1)} r(i)^k r(j)^k [Y(i)^k Y(j)^k]^0$$

with $k=2$. Like Elliott, we have not only the two-body $Q \cdot Q$ term, but also the $i=j$ single-particle term.

It is convenient to introduce the quantity $\bar{\chi} = 5b^4\chi/32\pi$ where b is the oscillator length parameter, such that $b^2 = \hbar/m\omega = 41.46/\hbar\omega$.

To evaluate the single-particle term we use the addition theorem

$$p_k(\cos \theta_{12}) = \frac{4\pi}{2k+1} \sum_{\mu} Y_{k,\mu}(1) Y_{k,\mu}^*(2)$$

and thus

$$\sqrt{5} [Y^2(i) Y^2(i)]^0 = \frac{5}{4\pi} P_2(1) = \frac{5}{4\pi}$$

The single-particle potential is then

$$U(r) = -\frac{\chi}{2} Q(i) \cdot Q(i) = -4\bar{\chi} \left(\frac{r}{b}\right)^4$$

The expectation values of $U(r)/\chi$ for the single-particle states, 0s, 0p, 0d, 1s, 0f, and 1p are, respectively, -15, -35, -63, -75, -99, and -119. What single-particle splitting $\epsilon_{0d} - \epsilon_{1s}$ is needed to get Elliott's SU(3) results? The best way to answer this is to give the formula for the SU(3) energy in the 1s-0d shell (in which the momentum terms are included):

$$E(\lambda\mu) = \bar{\chi} [-4(\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)) + 3L(L+1)]$$

For a rotational band, the $L=2$ - $L=0$ splitting.

The splitting due to the diagonal $Q \cdot Q$ interaction is $[-63 - (-75)]\chi = 12\bar{\chi}$. This is only 2/3 of the desired result. Where does the remaining 1/3 ($6\bar{\chi}$) come from?

The answer is that the missing part comes from the interaction of the valence particle with core. That is to say, in order to get Elliott's SU(3) results we must not only include the diagonal term, but also the particle-core interaction.

The expression for the particle-core interaction is

$$\delta\epsilon_j = -\chi \sum_{c, m_c} \langle \Psi_m^j(1) \Psi_{m_c}^c(2) | Q \cdot Q | \Psi_m^j(1) \Psi_{m_c}^c(2) - \Psi_m^j(2) \Psi_{m_c}^c(1) \rangle \quad (1)$$

where j and m represent all the quantum numbers of the valence nucleon (including isospin labels which have been suppressed) and c and m_c are the quantum numbers of a particle in the core.

With the above $Q \cdot Q$ interaction only the exchange term survives. The expression becomes

$$\delta\epsilon_j = \chi \sum_{c, m_c} \langle \Psi_m^j \Psi_{m_c}^c Q \Psi_{m_c}^c \Psi_m^j \rangle \quad (2)$$

We obtain $\delta\epsilon_{0d} - \delta\epsilon_{1s} = 6\bar{\chi}$. This is missing 1/3 of the splitting required to get Elliott's SU(3) results [1].

The above results for the 1s-0d shell are more general. In the 1p-0f shell the single-particle splitting required to get the SU(3) result is $\epsilon_{0f} - \epsilon_{1p} = 3(3 \times 4 = 1 \times 2)\bar{\chi} = 30\bar{\chi}$. Once again we only get 2/3 of this $20\bar{\chi}$ from the diagonal Q.Q term. The remaining $10\bar{\chi}$ comes from the interaction of the valence nucleon with the core (actually only the 0p shell in the core will contribute).

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Need for an isovector quadrupole term in the sum rule relating scissors mode excitations to B(E2) values

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Abstract

For a Q.Q interaction the energy-weighted sum rule for isovector orbital magnetic dipole transitions is proportional to the difference $\sum B(E2, \text{isovector}) - \sum B(E2, \text{isovector})$, not just to $\sum B(E2, \text{physical})$. This fact is important in ensuring that one gets the correct limit as one goes to nuclei, some of which are far from stability, for which one shell (neutron or proton) is closed. [S0556-2813(97)03208-1]

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Using the interaction $-\chi Q \cdot Q$, Zamick and Zheng [1,2] obtained a sum rule which relates the scissors mode excitation rate (i.e., the isovector orbital magnetic dipole excitation rate) to the electric quadrupole excitation rate. The isovector orbital magnetic dipole operator is $(\vec{L}_\pi - \vec{L}_\nu)/2$ [the isoscalar one is half the

total orbital angular momentum $\vec{L}/2 = (\vec{L}_\pi - \vec{L}_\nu)/2$. In more detail, sum rule reads

$$\sum_k (E_n - E_o) B(M1)_{n,\uparrow} = \frac{9\chi}{16\pi} \sum_i \{[B(E2, 0_1 \rightarrow 2_i)_{IS} - B(E2, 0_1 \rightarrow 2_i)_{IV}]\} \quad (1)$$

where $B(M1)_{n,\uparrow}$ is the value for the isovector orbital M1 operator ($g_{l\pi} = 0.5, g_{l\nu} = -0.5, g_{s\pi} = 0, g_{s\nu} = 0$) and the operator for the E2 transitions is $\sum_{protons} e_p r^2 Y_2 + \sum_{neutrons} e_n r^2 Y_2$ with $e_p = 1, e_n = 1$ for the isoscalar transition (IV). The above result also holds if we add a pairing interaction between like particles, i.e., between two neutrons and two protons. Our main objectives in this work are to clarify the role of the isovector B(E2) in the above formula and to compare the fermion and boson model approaches to scissors mode excitations.

The above work was motivated by the realization from many sources that there should be a relation between the scissors mode excitation rate and nuclear collectivity. Indeed, the initial picture by LoIudice and Palumbo [3] was of an excitation in a deformed nucleus in which the symmetry axis of the neutrons vibrated against that of the protons. In 1990-1991 contributions by the Darmstadt group [4,5], it was noted that the Sm isotopes, which undergo large changes in deformation as a function of mass number, the $B(M1)_{scissors}$, was proportional to $B(E2, 0_1 \rightarrow 2_1)$. The B(E2) in turn is proportional to the square of nuclear deformation, δ^2 .

The above energy-weighted sum rule of Zamick and Zheng [2] was attempt to obtain such a relationship microscopically using fermions rather than interacting bosons. To a large extent they succeeded, but there were some differences relative to [4,5]. Rather than being proportional to $B(E2, 0_1 \rightarrow 2_1)$, the proportionality factor was the difference in the summed isoscalar and summer isovector B(E2)'s. Now one generally expects the isoscalar B(E2), especially to the first 2^+ state, to be the most collective and much larger than the isovector B(E2). If the latter is negligible, then indeed one basically has the same relation between scissors mode excitations and nuclear collectivity, as empirically observed in the Sm isotopes.

However, the derivation of the above energy-weighted sum rule is quite general and should therefore hold (in the mathematical sense) in all regions, not just where the deformation is strong. To best illustrate the need for the isovector B(E2), consider a nucleus with a closed shell of neutrons or protons. In such a nucleus, and neglecting ground state correlations, the scissors mode excitation rate will vanish as one needs both open shell neutrons and protons to get a finite scissors mode excitation rate. However, if we have, say, an open shell of protons to get a finite scissors mode excitation rate. However, if we have, say an open shell of protons and a closed shell of neutrons, the $B(E2, 0_1 \rightarrow 2_1)$ can be quite substantial. Many vibrational nuclei are of such a type, and they have large B(E2)'s from the ground state, e.g., 20 Weisskopf units (W.u.).

However, in the above circumstances (closed neutron shell), the neutrons will not contribute to the B(E2) even if we give them an effective charge. But if only the protons contribute, it is clear that $B(E2, \text{isovector}) = B(E2, \text{isoscalar})$.

As an example, let us consider the even-even Be isotopes ${}^6\text{Be}$, ${}^8\text{Be}$, ${}^{10}\text{Be}$, and ${}^{12}\text{Be}$. In doing so, we go far away from the valley of stability, but this is in line with modern interests in radioactive beams.

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TABLE I. The values of $B(M1)_{\text{orbital}}$, $B(E2)_{\text{isoscalar}}$, $B(E2)_{\text{isovector}}$ for Be isotopes.

N_{nucleus}		$B(fm)^a$	$B(M1)_{\text{orbital}}$	$B(M1)_{\text{isoscalar}}$	$B(M1)_{\text{isovector}}$
${}^6\text{Be}$		1.553	0	15.63 ^b	15.63 ^b
${}^8\text{Be}$		1.597	0.637	73.54 ^c	6.47
${}^{10}\text{Be}$	$T = 1 \rightarrow T = 1$	1.635	0.0895	69.7	31.27
	$T = 1 \rightarrow T = 2$		0.149	0	3.20
${}^{12}\text{Be}$		1.669	0	20.85	20.85

^a $b^2 = 41.46/(\hbar\omega)$, $\hbar\omega = 45/A^{2/3} - 25/A^{1/3}$.

^bThe analytic expression in ${}^6\text{Be}$ is $B(E2) = (50/4\pi)b^4e_p^2$.

^cThe analytic expression in ${}^8\text{Be}$ is $B(E2) = (35/4\pi)b^4(ep + en)^2$.

Fayache, Sharma, and Zamick [6] have previously considered ${}^8\text{Be}$ and ${}^{10}\text{Be}$. The point was made that these two nuclei had about the same calculated $B(E2, 0_1 \rightarrow 2_1)$, but the isovector orbital B(M1)'s were significantly smaller in ${}^{10}\text{Be}$ than in ${}^8\text{Be}$. This went against the systematic that B(M1)orbital is proportional merely to B(E2). In detail, the calculated $B(M1, 0_1 \rightarrow 1)$ was $(2/\pi)\mu_N^2$ for ${}^8\text{Be}$ and in ${}^{10}\text{Be}$ was $(9/32\pi)\mu_N^2(T = 1 \rightarrow T = 1)$ and $(15/32\pi)\mu_N^2(T = 1 \rightarrow T = 2)$. Thus the ratio of isovector orbital B(M1)'s is ${}^{10}\text{Be}/{}^8\text{Be} = 3/8$.

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Seniority Conservation and Seniority Violation in the $g_{9/2}$ shell

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Abstract

The $g_{9/2}$ shell of identical particle is the first one for which one can have seniority-mixing effects. We consider three interactions: a delta interaction that conserves seniority, a quadrupole-quadrupole (Q.Q) interaction that does not, and a third one consisting of two-body matrix elements taken from experiment (${}^{98}\text{Cd}$) that also leads to some seniority mixing. We deal with proton holes relative to a $Z=50$, $N=50$ core. One surprising

result is that, for a four-particle system with total angular momentum $I=4$, there is one state with seniority $\nu=4$ that is an eigenstate of any two-body interaction-seniority conserving or not. The other two states are mixtures of $\nu=2$ and $\nu=4$ for the seniority-mixing interactions. The same thing holds true for $I=6$. Another point of interest is that, in the single-j shell approximation, the splitting $\Delta E = E(I_{max}) - E(I_{min})$ are the same for three and five particles with a seniority conserving interaction (a well-known result), but are equal and opposite for a Q.Q interactions. The $Z=40$, $N=40$ core plus $g_{9/2}$ neutrons (Zr isotopes) is also considered, although it is recognized that the core is deformed.

III. SPECIAL BEHAVIORS FOR $I=4+$ AND $6+$ STATES OF THE $g_{9/2}$ CONFIGURATION

For a system of four identical nucleons in the $g_{9/2}$ shell, the possible seniorities are $\nu=0, 2$, and 4 , with $\nu=0$ occurring only for a state of total angular momentum $I=0$. There is also a $\nu=4$ state with $I=0$.

For $I=4$ and 6 , we can have three states, one with seniority $\nu=2$ and two with seniority $\nu=4$. For the two $\nu=4$ states we have at hand, we can construct different sets of $\nu=4$ states by taking linear combinations of the original ones. If the original ones are $(4)_1$ and $(4)_2$, we can form

$$(4)_A = a(4)_1 + b(4)_2, \quad (2)$$

$$(4)_B = -b(4)_1 + a(4)_2$$

with $a^2 + b^2 = 1$. The set $(4)_A, (4)_B$ is as valid as the original set.

However, we here note that if we perform a matrix diagonalization with any two-body interaction-seniority conserving or not-one state emerges which does not depend on what the interaction is. The other two states are, in general, mixtures of $\nu=2$ and $\nu=4$ which do depend on the interaction. The values of the coefficients of fractional parentage (cfp's) of this unique state of seniority 4 as shown in Table I. The states of $J_o \neq 4.5$ all have seniority $\nu=4$ state there is no admixture of states with $J_o = j = 9/2$, be they $\nu=1$ or $\nu=3$. Again, no matter what two-body interaction is used, this $I=4$ state remains a unique state.

TABLE I: A unique $J=4, \nu=4$ SFP for $j=9/2$

J_o	$(j^3 J_o j j^4 I=4, \nu=4)$
1.5	0.1222
2.5	0.0548
3.5	0.6170
4.5 ($\nu=1$)	0.0000
4.5 ($\nu=3$)	0.0000
5.5	-0.4043
6.5	-0.6148
7.5	-0.1597
8.5	0.1853

Amusingly, this state does not appear in the compilation of seniority-classified cfp's of Bayman and Lande [20] or de Shalit and Talmi [5]. We should empha-

size that, although different, the Bayman-Lande cfp's are perfectly correct (as are the ones of de Shalit and Talmi, whose cfp's are also different from those of Bayman and Lande [20]). But then, why do they not obtain the unique state that we have shown above? Bayman and Lande use group theoretical techniques to obtain the cfp's diagonalizing the following Casimir operator for $Sp(2j+1)$:

$$G(Sp_{2j+1}) = \frac{1}{2j+1} \sum_{\text{odd } k=1}^{2j} (-1)^k (2k+1)^{3/2} [U^k U^k]_0^0 \quad (3)$$

where $U_q^k \equiv \sum_{i=1}^N U_q^k(i)$ and U is the Racah unit tensor operator

$$\langle \Psi_{m'}^{j'} | U_q^k | \Psi_m^j \rangle = \delta_{jj'} (k j q m | j' m') \quad (4)$$

The two seniority $\nu=4$ states are degenerate with such an interaction and, since there is no seniority mixing, we can have arbitrary linear combinations of the 4^+ states. Only by using an interaction which removes the degeneracy and violates seniority, do we learn about the special state in Table I.

IV. THE ENERGY SPLITTING $E(I_{\text{max}}) - E(I_{\text{min}})$ WITH A Q.Q INTERACTION

A well-known result for identical particles in a single j shell is that, if one uses a seniority-conserving interaction, then the relative spectra of states of the same seniority are independent of the number of particles [5-7]. Thus, for $n=3$ and $n=5$, the seniority $\nu=3$ states have the same relative spectrum; for $n=2, 4$, and 6 , the seniority $\nu=2$ states have the same spectrum. These results hold, in particular, for the delta interaction used here.

Now the Q.Q interaction does not conserve seniority and the above results do not hold. However, we have noticed an interesting result for $n=3$ and $n=5$. Consider the splitting $E(I_{\text{max}}) - E(I_{\text{min}})$, $\nu=3$, where for $g_{9/2}$, $I_{\text{max}} = 21/2$ and $I_{\text{min}} = 3/2$. For a seniority-conserving interaction, $\Delta E(n=5) = \Delta E(n=3)$, where for a Q.Q interaction, $\Delta E(n=5) = -\Delta E(n=3)$. This will be discussed quantitatively later.

X. THE $E(I_{\text{max}}) - E(I_{\text{min}})$ SPLITTING FOR $n=3$ AND $n=5$: ^{97}Ag VERSUS ^{95}Rh AND ^{83}Zr VERSUS ^{85}Zr

As mentioned in a previous section, the splitting $\Delta E = E(I_{\text{max}} = 21/2^+) - E(I_{\text{min}} = 3/2^+)$ is the same for three particles as it is for five particles (or three holes and five holes) if one has a seniority-conserving interaction. However, for a pure Q.Q interaction, we have $\Delta E(n=5) = -\Delta E(n=3)$.

Using the $V(^{98}\text{Cd})$ interaction for ^{97}Ag and ^{95}Rh , we find

$$\Delta E(n=3) = 0.77058 \text{ MeV}$$

$$\Delta E(n=5) = 0.87818 \text{ MeV}$$

They are both positive, an indication that the seniority-conserving delta interaction is much more important than the seniority-violating Q.Q interaction.

Talmi had previously concluded, from an analysis of h11/2 nuclei with a closed shell of neutrons (N=82), that seniority conservation held to a high degree [7,31].

Unfortunately, for the $g_{9/2}$ nuclei that we are here considering (^{92}Tc , ^{95}Rh , ^{97}Ag , as well as the zirconium isotopes ^{83}Zr , ^{85}Zr , ^{87}Zr), although the high spin states including $I = 21/2^+$ have been identified, the $I = 3/2^+$ states in ^{97}Ag , ^{95}Rh , and ^{93}Tc , as well as for Zr isotopes.

For ^{83}Zr and ^{85}Zr , with a fitted interaction (despite misgivings of using a single j model space), we find for $\Delta E = E(I_{max}) - E(I_{min})$.

$$\Delta E(^{93}\text{Zr}) = 0.48742 \text{ MeV}$$

$$\Delta E(^{95}\text{Zr}) = -0.59355 \text{ MeV}$$

They have opposite signs, which shows that for these fitted interactions the Q.Q interaction is much more important for this case-neutrons beyond a Z=40, N=40 core.

But it should be emphasized that the $I = 3/2^+$ state is not part of the fit because it has not been identified experimentally. If more levels were known in the Zr isotopes, and in particular the low spin level $I = 3/2^+$ (but also $5/2^+$ and $1/2^+$), then the picture might change. We strongly urge that experimental work be done on all the nuclei considered here in order to locate the missing states, especially $I = 3/2_1^+$ and also $5/2_1^+$.

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New relations for coefficients of fractional parentage-The Redmond recursion formula with seniority

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Abstract

We find a relationship between coefficients of fractional parentage (cfp) obtained on the one hand from the principal-parent method and on the other hand from a seniority classification. We apply this to the Redmond formula which relates $n \rightarrow n+1$ cfp's to $n-1 \rightarrow n$ cfp's where the principal-parent classification is used. We transform this to the seniority scheme. Our formula differs from the Redmond formula in as much as we have a sum over the possible seniorities for the $n \rightarrow n+1$ cfp's, whereas Redmond has only one term. We show that there are useful applications of both the principal-parent and the seniority classification.

A recursion formula for cfp's due to Redmond [1] is presented in the books of de Shalit and Talmi [2] on p. 528, and Talmi [3] on p. 274. It can be written as follows

$$(n+1) [j^n (\alpha_0 J_0) j J | \{ j^{n+1} [\alpha_0 J_0] J] [j^n (\alpha_1 J_1) j J | \{ j^{n+1} [\alpha_0 J_0] J] =$$

$$\delta_{\alpha_1 \alpha_0} \delta_{J_1 J_0} + n(-1)^{J_0+J_1} \sqrt{(2J_0+1)(2J_1+1)} \sum_{\alpha_2 J_2} \begin{matrix} J_2 & j & J_1 \\ J & j & J_0 \end{matrix}$$

$$\times [j^{n-1} (\alpha_2 J_2) j J_0 | \{ j^n \alpha_0 J_0] [j^{n-1} (\alpha_2 J_2) j J_1 | \{ j^n \alpha_1 J_1] \quad (7)$$

In the above, square bracket designates the principal parent used to calculate the cfp. Actually, the principal parent sometimes loses its significance because in some cases more than one principal parent can yield the same cfp. In tables of cfp's, the principal parent is usually not listed. The quantities in parentheses ($\alpha_o J_o$) are listed. The cfp with ($\alpha_o J_o$) is the probability amplitude that a system of $(n+1)$ identical particles with quantum numbers ($\alpha_o J_o$) and a single nucleon.

2. Relation between principal-parent cfp's and those in the seniority scheme We here note a relationship between the overcomplete set of principal-parent coefficients of fractional parentage and those with the seniority classification

$$[j^n (\nu_0 J_0) j J | \{ j^{n+1} [\nu_0 J_0] J] [j^n (\nu_1 J_1) j J | \{ j^{n+1} [\nu_0 J_0] J] =$$

$$\sum_{\nu} [j^n (\nu_0 J_0) j J | \{ j^{n+1} J \nu] [j^n (\nu_1 J_1) j J | \{ j^{n+1} J \nu] \quad (8)$$

In the left-hand side above, the first principal parent is formed by adding the $(n+1)$ th nucleon to an n -nucleon to an n -nucleon antisymmetric system with good seniority and angular momentum ($\nu_o J_o$), then coupling the combined system to a total angular momentum J , and then antisymmetrizing and normalizing the total wave function. On the right-hand side, the sum over ν is a sum over all the possible seniorities of the combined $(n+1)$ system and, for a given seniority, over all states with that seniority.

A proof of the above result will be given in Appendix A.